EYIB
EDUCATIONAL YIELD INSIGHTS & BREAKTHROUGHS
TA'LIMGA OID TUSHUNCHALAR VA YUTUQLAR
ISSN:3060-4648
Vol.3 №3
https://eyib.uz

TESKARI MATRITSANI HISOBLASH USULLARI VA UNGA MOS DASTUR YARATISH HAQIDA

Raximova Feruza Saidovna	*Muhammad al-Xorazmiy nomidagi TATU katta o'qituvchisi*
Tursunaliyev Ozodbek Bahrom o'g'li	*Muhammad al-Xorazmiy nomidagi TATU talabasi*
Safarboyev Sarvar Rashid o'g'li	*Muhammad al-Xorazmiy nomidagi TATU talabasi*
Iskandarova Dilafruz Sharofaddin qizi	*Muhammad al-Xorazmiy nomidagi TATU talabasi*
Xusnutdinov Azimbek Jahongir o'g'li	*Muhammad al-Xorazmiy nomidagi TATU talabasi*
Isroilova Gulhayo Akmal qizi	*Muhammad al-Xorazmiy nomidagi TATU talabasi*

Annotatsiya

Ushbu ishda matritsaga teskari matritsani topishning noan'anaviy usuli ko'rsatilgan. Bu usulda hisoblash bo`yicha nazatiy ma'lumotlar keltirilgan. Teskari matritsani klassik usulda va noan'anaviy usulda yechilgan va yechimlari taqqoslab ko`rsatilgan. Bu usulda teskari matritsani topish qulayliklari ko`rsatilgan. Shuningdek, teskari matritsani hisoblash dasturi keltirilgan. Dasturda matrisa elementlarini ixtiyoriy berilganda ham teskari matritsani topishning afzalliklari ko`rsatilgan.

Kalit so'zlar: A kvadrat, matritsa, determinant, algebraik to'ldiruvchi, algebra nazariyasi, minorlar usuli.

Biz bilamizki, agar A kvadrat matritsaning determinanti noldan farqli bo'lsa, ya'ni $A \neq 0$ bo`lsa, matritsa xosmas matritsa bo`ladi. Aks holda matritsa xos matritsa hisoblanadi. Matritsaning teskarisi to'g'ri hisoblanganini $A \cdot A^{-1} = E$ tenglik bilan tekshirish mumkin. Bu yerda E matritsa A matritsa o'lchovi bilan bir xil o'lchovli birlik matritsadir.

Xosmas matritsa uchun yagona teskari matritsa mavjud va quyidagi formula bilan hisoblanadi:

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}.$$

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ matritsa uchun teskari A^{-1}matritsani klassik usulda topamiz. Buning uchun dastlab A matritsaning determinantini hisoblaymiz.

Determinant quyidagi formula asosida hisoblanadi:

$$|A| = (a_{11} * a_{22} * a_{33} + a_{12} * a_{23} * a_{31} + a_{13} * a_{21} * a_{32}) -$$
$$-(a_{13} * a_{22} * a_{31} + a_{12} * a_{21} * a_{33} + a_{11} * a_{23} * a_{32})$$

Determinant nolga teng emasligi aniqlangach, berilgan matritsaning teskari matritsasini topish mumkin. Agar $\det A \neq 0$ bo‘lsa, u holda algebraik to‘ldiruvchilarni quyidagi formulalar bilan hisoblaymiz:

$$A_{11} = (-1)^{1+1} * \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22} * a_{33} - a_{23} * a_{32}$$
$$A_{12} = (-1)^{1+2} * \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = -(a_{21} * a_{33} - a_{23} * a_{31})$$
$$A_{13} = (-1)^{1+3} * \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{21} * a_{32} - a_{22} * a_{31}$$
$$A_{21} = (-1)^{2+1} * \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = a_{12} * a_{13} - a_{23} * a_{32}$$
$$A_{22} = (-1)^{2+2} * \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = a_{11} * a_{33} - a_{13} * a_{31}$$
$$A_{23} = (-1)^{2+3} * \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} = a_{11} * a_{32} - a_{12} * a_{31}$$
$$A_{31} = (-1)^{3+1} * \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} = a_{12} * a_{23} - a_{13} * a_{22}$$
$$A_{32} = (-1)^{3+2} * \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = a_{11} * a_{23} - a_{13} * a_{21}$$
$$A_{33} = (-1)^{3+3} * \begin{vmatrix} a_{11} & a_{12} \\ a_{22} & a_{22} \end{vmatrix} = a_{11} * a_{22} - a_{12} * a_{22}$$

Agar $\det A \neq 0$ bo‘lsa, u holda teskari matritsa topishning boshqa usulini ko‘rib chiqamiz.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Bu usulda hisoblash uchun matritsaning birinchi ikkita satri pastga ko‘chirib yoziladi va boshqa B matritsaga ko‘chiriladi:

$$B = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$$

So‘ng birinchi ikkita ustun o‘ng tomonga qo‘shiladi:

$$B = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \\ a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \end{pmatrix}$$

Keyingi bosqichda birinchi satr va birinchi ustun olib tashlanadi, natijada 2×2 determinantlarni hisoblash uchun qulay jadval hosil bo‘ladi.

$$B = \begin{pmatrix} a_{22} & a_{23} & a_{21} & a_{22} \\ a_{32} & a_{33} & a_{31} & a_{32} \\ a_{12} & a_{13} & a_{11} & a_{12} \\ a_{22} & a_{23} & a_{21} & a_{22} \end{pmatrix}$$

Mazkur bosqichda berilgan 3×3 o‘lchamli matritsaning har bir elementi uchun mos minorlar aniqlanadi. 2×2 o‘lchamli minorlarni quyidagi tartibda topamiz:

M_{ij}^{mk} minor sifatida bu matritsaning i-ustuni va j-ustuni hamda m-satri va k-satrlari kesishmasidan hosil bo‘ladigan 2×2 o‘lchamli yordamchi matritsaning determinantlarini hisoblaymiz va mos o`rinlarga qo`yib chiqamiz.

$$M_{12}^{12} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \qquad M_{23}^{12} = \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} \qquad M_{34}^{12} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$M_{12}^{23} = \begin{vmatrix} a_{32} & a_{33} \\ a_{12} & a_{13} \end{vmatrix} \qquad M_{23}^{12} = \begin{vmatrix} a_{33} & a_{31} \\ a_{13} & a_{11} \end{vmatrix} \qquad M_{34}^{12} = \begin{vmatrix} a_{31} & a_{32} \\ a_{11} & a_{12} \end{vmatrix}$$

$$M_{12}^{34} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \qquad M_{23}^{12} = \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \qquad M_{34}^{12} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$B = \begin{pmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ \begin{vmatrix} a_{32} & a_{33} \\ a_{12} & a_{13} \end{vmatrix} & \begin{vmatrix} a_{33} & a_{31} \\ a_{13} & a_{11} \end{vmatrix} & \begin{vmatrix} a_{31} & a_{32} \\ a_{11} & a_{12} \end{vmatrix} \\ \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

Chiziqli algebra nazariyasiga ko‘ra, n-tartibli kvadrat matritsaning teskari matritsasi quyidagi formula asosida aniqlanadi:

$$A^{-1} = \frac{1}{|A|} B^T$$

Quyidagi misolni qaraymiz. Bunda 3×3 o‘lchamli matritsaning teskari matritsaning teskarisini topish talab qilingan.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix}$$

Mazkur masalada teskari matritsa avvalo minorlar usuli orqali topiladi, so‘ngra natija algebraik toldiruvchilar yordamida an’anaviy usul yordamida tekshiriladi.

Berilgan 3×3 o‘lchamli determinantning 1- va 2ustunlarni determinantning o`ng tomoniga yozib, Sarrius usuli yordamida aniqlanadi. Ushbu qoida faqat 3×3 o`lchamli determinantlar uchun qo‘llaniladi va determinantni hisoblash jarayonini soddalashtiradi. Berilgan matritsaga mos determinantni hisoblaymiz:

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{vmatrix} = (50 + 96 + 84) - (105 + 48 + 80) = -3$$

Demak, $|A| = -3 \neq 0$ shuning uchun matritsa teskari matritsaga ega.

Dastlabki bosqichda matritsani kengaytiramiz. Minorlarni qulay hisoblash maqsadida matritsaning birinchi ikkita satri pastga ko‘chirib yoziladi:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

So‘ng birinchi ikkita ustun o‘ng tomonga qo‘shiladi:

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 10 & 7 & 8 \\ 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \end{pmatrix}$$

Keyingi bosqichda birinchi satr va birinchi ustun olib tashlanadi, natijada 2×2 determinantlarni hisoblash uchun qulay jadval hosil bo‘ladi.

$$\begin{pmatrix} 5 & 6 & 4 & 5 \\ 8 & 10 & 7 & 8 \\ 2 & 3 & 1 & 2 \\ 5 & 6 & 4 & 5 \end{pmatrix}$$

Keyingi bosqichda 2×2 determinantlarni hisoblanadi, ya’ni minorlar aniqlanadi.

Minorlarni hisoblashni umumiy holda quyidagicha amalga oshiriladi:

$$M_{12}^{12} = \begin{vmatrix} 5 & 6 \\ 8 & 10 \end{vmatrix} = 50 - 48 = 2, \quad M_{23}^{12} = \begin{vmatrix} 6 & 4 \\ 10 & 7 \end{vmatrix} = 42 - 40 = 2, \quad M_{34}^{12} = \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = 32 - 35 = -3$$

$$M_{12}^{23} = \begin{vmatrix} 8 & 10 \\ 2 & 3 \end{vmatrix} = 24 - 20 = 4, \quad M_{23}^{23} = \begin{vmatrix} 10 & 7 \\ 3 & 1 \end{vmatrix} = 10 - 21 = -11, \quad M_{34}^{23} = \begin{vmatrix} 7 & 8 \\ 1 & 2 \end{vmatrix} = 14 - 8 = 6$$

$$M_{12}^{34} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = 12 - 15 = -3,$$

$$M_{23}^{34} = \begin{vmatrix} 3 & 1 \\ 6 & 4 \end{vmatrix} = 12 - 6 = 6, \quad M_{34}^{34} = \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = 5 - 8 = -3$$

Natijada quyidagi 3×3 minorlarning natijaviy matritsasi hosil bo‘ladi:

$$B = \begin{pmatrix} 2 & 2 & -3 \\ 4 & -11 & 6 \\ -3 & 6 & -3 \end{pmatrix}$$

Topilganlardan foydalanib, teskari matritsani hisoblaymiz. Natija qiymatlarni o`rniga qo‘yib, berilgan matritsaning teskari matritsasini topish mumkin.

$$A^{-1} = \frac{1}{|A|} B^T$$

B matritsani transponirlaymiz:

$$B^T = \begin{pmatrix} 2 & 4 & -3 \\ 2 & -11 & 6 \\ -3 & 6 & -3 \end{pmatrix}$$

Transponirlangan B matritsa asosida teskari matritsani hisoblab chiqamiz.

$$A^{-1} = \frac{1}{|A|} B^T = \frac{1}{-3} \begin{pmatrix} 2 & 4 & -3 \\ 2 & -11 & 6 \\ -3 & 6 & -3 \end{pmatrix}$$

Algebraik to‘ldiruvchilar yordamida an’anaviy usul bilan teskari matritsani to‘g‘ri yoki noto‘g‘ri ekanini tekshiramiz. Biz bilamizki, klassik usulda n-tartibli kvadrat matritsaning teskari matritsasi quyidagi ketma-ketlik asosida aniqlanadi. Teskari matritsa topishning umumiy formulasi quyidagicha bo‘ladi:

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

Bu yerda a_{ij} elementning mos algebraik to‘ldiruvchisi quyidagicha topiladi:

$$A_{ij} = (-1)^{i+j} * M_{ij}$$

Algebraik to ‘ldiruvchilarni topib, matritsaga qo‘yib hisoblab ko‘ramiz:

$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix}$, $|A| = -3$

$A_{11} = 50 - 48 = 2$ $A_{21} = -(20 - 24) = 4$ $A_{31} = 12 - 15 = -3$

$A_{12} = -(40 - 42) = 2$ $A_{22} = 10 - 21 = -11$ $A_{32} = 6 - 12 = 6$

$A_{13} = 32 - 35 = -3$ $A_{23} = -(8 - 14) = 6$ $A_{33} = 5 - 8 = -3$

Teskari matritsani topib, dastlabki usulda topilgan matritsa bilan taqqoslaymiz va natijaning bir xilligini ko`ramiz:

$$A^{-1} = \frac{1}{-3}\begin{pmatrix} 2 & 4 & -3 \\ 2 & -11 & 6 \\ -3 & 6 & -3 \end{pmatrix}$$

Ushbu dastur kodi yuqorida keltirilgan minorlar usuli asosida berilgan 3×3 matritsaning teskari matritsasini hisoblab beradi.

```
def print_matrix(M, title=""):

    if title:

        print(title)

    for row in M:

        print("  ".join(f"{x:8.4f}" for x in row))

    print()




def determinant(A):

    n = len(A)



    if n == 1:

        return A[0][0]
```

```
    if n == 2:
        return A[0][0] * A[1][1] - A[0][1] * A[1][0]

    det = 0
    for j in range(n):
        minor = [row[:j] + row[j+1:] for row in A[1:]]
        det += ((-1) ** j) * A[0][j] * determinant(minor)

    return det


def cofactor_matrix(A):
    n = len(A)
    C = [[0] * n for _ in range(n)]

    for i in range(n):
        for j in range(n):
            minor = [
                row[:j] + row[j+1:]
                for k, row in enumerate(A) if k != i
            ]
```

```
            C[i][j] = ((-1) ** (i + j)) * determinant(minor)

    return C



def transpose(A):
    return [list(row) for row in zip(*A)]



def inverse_matrix(A):
    detA = determinant(A)

    if detA == 0:
        raise ValueError("Determinant 0 ga teng — teskari matritsa yo‘q")

    C = cofactor_matrix(A)
    adjA = transpose(C)

    n = len(A)
    invA = [[adjA[i][j] / detA for j in range(n)] for i in range(n)]

    return detA, invA
```

```
A = [
    [1, 2, 3],
    [4, 5, 6],
    [7, 8, 10]
]

print_matrix(A, "A matritsa:")

detA, A_inv = inverse_matrix(A)

print(f"|A| = {detA}\n")
print_matrix(A_inv, "Teskari matritsa:")
```

Dastur kodi bajarilishi natijasida quyidagi qiymatlar hosil bo‘ldi.

```
A matritsa:
  1.0000    2.0000    3.0000
  4.0000    5.0000    6.0000
  7.0000    8.0000   10.0000

|A| = -3

Teskari matritsa:
 -0.6667   -1.3333    1.0000
 -0.6667    3.6667   -2.0000
  1.0000   -2.0000    1.0000


Process finished with exit code 0
```

Teskari matritsa topishning noan'anaviy usuliga dastur tuzilishining afzallik tomoni shundaki, undan foydalanib, elementlari turlicha bo'lgan matritsalarga teskari matritsalarni osongina topish mumkin.

Matematik fanlar mavzularini dasturlovchi talabalarga o'qitishdagi bir necha yillik tajribalar shuni ko`rsatadiki, mavzuni o'tishda talabalarga dasturlash fanidan foydalanib o'zlashtirish bo`yicha yo`nalish berish talabalarning mavzuni o'rganishga bo`lgan qiziqishlarini ancha orttiradi. Shu bilan birga dars samaradorligini sezilarli darajada o`stiradi.

Adabiyotlar:

[1] Стренг Г. Линейная алгебра и ее применения. М.:Мир 2020. 454с.

[2] Bakhtiyar Rakhimov, Feruza Rakhimova, Atabek Saidov, Zarina Saidova. Analysis and modeling of digital solution in medical database management. ITM WEB of Conferences 72, 03002 (2025), HMMOCS-III 2025, https://doi.org/10.1051/itmconf/20257203002 6 p. https://www.itm-conferences.org/articles/itmconf/abs/2025/03/itmconf_hmmocs-III2024_03002/itmconf_hmmocs-III2024_03002.html

[3] Bakhtiyar Rakhimov, Ozodov Ravshonbek, Feruza Rakhimova, Atabek Saidov, Zarina Saidova. Metadata of the chapter that will be visualized in SpringerLink. Springer Nature Switzerland AG 2025 P.S.Stanimirovic et al.(Eds.): LNNS 1481, pp.1-10, 2025. https://doi.org/10.1077/978-3-031-9549-2_9.

[4] Marat Karimov, Feruza Rakhimova. Modeling of Groundwater Flow in a Multilayer Porous Medium Based on a Nonlinear Mathematical Model. Fourth International Conference on Digital Technologies, Optics, and Materials Science (DTIEE 2025). – SPIE, 2025. – Т. 13662. – С. 136-141.0277-786X, 136620J-1. https://doi.org/10.1117/12.3072569

[5] Raximova Feruza Saidovna, Islamova Odila Abduraimovna, Chay Zoya Sergeevna, Fayzullayeva Shahlo Alisherovna, Madatova Zuxra Abdiraximovna. "Talabalarga qutb koordinatalar sistemasida funksiyalar grafigini chizishni dasturlardan foydalanib o'rgatish". "FIZIKA, MATEMATIKAVA SUN'IY INTELLEKT TEXNOLOGIYALARINING DOLZARB MUAMMOLARI" XALQARO ILMIY NAZARIY ANJUMAN materiallari (may 16-17, 2025). 173-175 bet

[6] Rakhimov, B.S., Rakhimova, F.B., Sobirova, S.K. Modeling database management systems in medicine, Journal of Physics: Conference Series, 2021, 1889(2), 022028

[7] Rakhimov, Bakhtiyar; Ismoilov, Olimboy; ,Management systems for modeling medical database, AIP Conference Proceedings, 2432, 1, 060031, 2022, AIP Publishing LLC

[8] Allaberganov, Odilbek R; Rakhimov, Bakhtiyar S; Sobirova, Sabokhat K; Rakhimova, Feroza B; Saidov, Atabek B; ,Problem for medical system with infinite zone potential in the half line,AIP Conference Proceedings,2647,1,050025,2022,AIP Publishing LLC

[9] Saidovich, Rakhimov Bakhtiyar; Gafurovich, Bekchanov Bakhtiyar; Alimovna, Jumaniyazova Tupajon; Bakhtiyarovich, Saidov Atabek; ,Processor Architectures in Data Base Problems,2023,PROCEDIA PUBLISH GROUP